# Spectral band passes for a high precision satellite sounder

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Atmospheric temperature soundings with significantly improved vertical resolution can be obtained from carefully chosen narrow band-pass measurements in the 4.3-$\mu$m band of $CO_2$ by taking advantage of the variation of the absorption coefficients, and thereby the weighting functions, with pressure and temperature. A set of channels has been found in the 4.2-$\mu$m region that is capable of yielding about 2-km vertical resolution in the troposphere. The concept of a complete system is presented for obtaining high resolution retrievals of temperature and water vapor distribution, as well as surface and cloud top temperatures, even in the presence of broken clouds.

## Introduction

Numerical circulation models have already developed to the point that the number of tropospheric layers is greater by more than a factor of 2 than the number of sounding levels in current or presently scheduled sounders.[1,2]

A limiting constraint on the vertical resolution of retrieved soundings is the width of the weighting functions, defined as $-d\tau/dlnp$, where $\tau$ is the channel-averaged transmittance and $p$ is pressure. The principal purpose of this paper is to show that sufficient narrowing of the weighting functions to obtain the required vertical fine structure in the retrievals can be obtained by a careful choice of channels to utilize the dependence of the absorption coefficients on pressure and temperature.

We will begin by a background discussion of the advantages of measurements in the line wings over broadband measurements from the point-of-view of narrowness of temperature-independent weighting functions, then show that efficient utilization of the temperature dependence can narrow them even further. Finally, we will describe an all-weather sounding system capable of obtaining high resolution retrievals of temperature and humidity distribution.

## Background

The governing equation for the emitted flux reaching the top of a plane-parallel stratified atmosphere overlying a black surface at pressure $p_s$ is

$$I_j = \int_{p_s}^{0} B_j[T(p)][d\tau_j(p,0)/dlnp]dlnp + B_j[T(p_s)]\tau_j(p_s,0), \quad (1)$$

where $B_j(T,\nu_j)$ is the blackbody radiance in a channel centered at frequency $\nu_j$.

If the channel has an effective band pass $D_j$ and contains a random array of lines of individual equivalent widths $W_{ji}$, the average transmittance is[3]

$$\tau_j(p,0)_{\text{lines}} = \exp\left[-\sum_i W_{ji}(p,0)/D_j\right] \quad (2)$$

to the extent that the lines in $D_j$ are representative of those in a broader band pass. If, further, the lines have a Lorentz shape and their intensities $S_{ji}$ and half widths $\alpha_{ji}$ are independent of temperature and the absorbing gas is uniformly mixed in the atmosphere with volume fraction $q$, the individual equivalent widths have the value[4]

$$W_{ji}(p,0) = 2\sqrt{\pi}(\alpha_{0_{ji}}p/p_0)\Gamma(\eta_{ji} + ½)/\Gamma(\eta_{ji}), \quad (3)$$

where $\alpha_{0_{ji}}$ is the halfwidth at reference pressure $p_0$, and

$$\eta_{ji} = (qS_{ji}p_0)/(2\pi\alpha_{0_{ji}}\rho_0 g) \quad (4)$$

is independent of height, with $\rho_0$ being the reference density used to define $S_{ji}$ and $g$ the acceleration of gravity.

With the substitution of Eq. (3), Eq. (2) becomes

$$\tau_j(p,0)_{\text{lines}} = \exp(-a_j p), \quad (5)$$

where

$$a_j = \frac{2\sqrt{\pi}}{p_0 D_j}\sum_i \alpha_{0_{ji}}\Gamma(\eta_{ji} + ½)/\Gamma(\eta_{ji}) \quad (6)$$

is a constant, which approaches proportionality to

$$\sum_i (S_{ji}\alpha_{0_{ji}})^{1/2}$$

as the $\eta_{ji}$'s increase.

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Equation (5) is a reasonable representation of the form of the transmittances in those parts of the $CO_2$ spectra where the temperature dependence of the line intensities is small. The effect of strong dependence of intensities on temperature will be considered later in this paper.

For transmittance of the form of Eq. (5), the weighting function of $B_j$ is

$$-(d\tau/dlnp)_{\text{lines}} = a_j p \exp(-a_j p). \tag{7}$$

Equation (7) reaches its peak value at $a_j p = 1$. The ratio of the two pressures at which the weighting functions have half of the peak value can be computed to be $11.55 = \exp(2.45)$. Thus the width at half-peak of the weighting function is of the order of 2½ scale heights. Almost 28% of the area of the weighting function vs $lnp$ curve falls outside the half-peak levels.

It has long been recognized that increased vertical resolution of the retrieved thermal structure can be accomplished by narrowing the weighting functions, and two procedures for accomplishing this have been introduced. One is by limb-scanning, which is rarely applicable to the troposphere, and the other by selective chopping, which also has limited tropospheric applicability. The purpose of selective chopping[5,6] is to sound in line wings, where the transmittance at any frequency $\nu_j$ has the form

$$\tau_j(p,0)_{\text{wing}} = \exp(-b_j p^2), \tag{8}$$

where $b_j$ is, like $a_j$, a function of the line intensities, line widths at a reference pressure, and $CO_2$ mixing ratio, again all assumed constant.

The monochromatic weighting function in this case,

$$-(d\tau/dlnp)_{\text{wing}} = 2b_j p^2 \exp(-b_j p^2), \tag{9}$$

has half the width at half peak as that of Eq. (7), i.e., 1¼ scale heights. Model calculations show that the weighting function for a finite channel is broadened only slightly provided that the coefficient $b_j(\nu)$ does not vary greatly as a function of frequency across the channel. The factor-of-2 decrease in the weighting function width obtained by selective measurements in line wings can be more closely achieved in practice by narrowband measurements at discrete frequencies in the wings. This procedure, as we shall see, has other advantages.

Kaplan[7] has pointed out that soundings in high frequency bands, e.g., the 4.3-$\mu$m $CO_2$ band rather than the 15-$\mu$m band, also narrow the region of the troposphere from which the flux comes because of the stronger temperature dependence of $B_j$. The flux contribution per $dlnp$ is the integrand of Eq. (1), i.e., $B_j d\tau_j/dlnp$, which gives a better representation of the vertical resolving capability than $d\tau_j/dlnp$ itself. The high frequency branch of the 4.3-$\mu$m $CO_2$ band is also relatively free of isotope and hot lines, so that enough frequency intervals can be found in the line wings to sound the atmosphere from the surface to the 20-mb level.

Appropriate frequency intervals were selected by examining synthetic atmospheric spectra generated in the 4.3-$\mu$m region using the McClatchey *et al.*[8] line parameters. Our method of calculating transmittance is described elsewhere.[9] Figure 1 shows an example of synthetic spectra with the positions of the principal lines taken from Oberly *et al.*[11] and a comparison with measurements made by Reinhard Beer of Jet Propulsion Laboratory with a high-resolution interferometer. Figure 2 shows the set of weighting functions calculated for a standard atmosphere using triangular response functions with channel widths $\Delta\nu = 0.2$ cm$^{-1}$ at half height centered at the selected frequencies shown. Also shown are the curves of $Bd\tau/dlnp$, which are appreciably narrower in the troposphere but somewhat broader in the tropopause region and in the stratosphere.

Because of temperature effects to be discussed below, slightly broader tropospheric weighting functions are obtained, surprisingly, if the channel widths are decreased by a factor of 2.

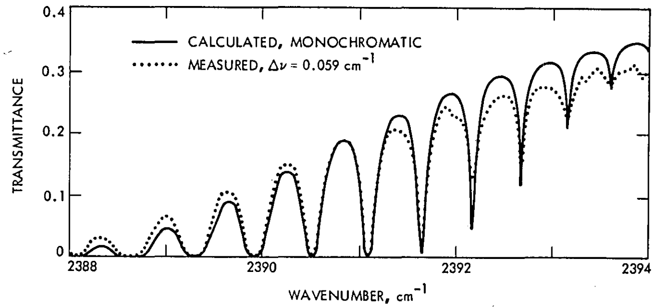


Fig. 1. Comparison of calculated monochromatic transmittance with that obtained from Beer's measurements at 1.53 air masses, with $p_s$ = 793 mb and $T_s$ = 288.5 K. Most of the differences can be accounted for by air mass uncertainties. See Ref. 10 for detailed discussion and comparison over a larger frequency range.

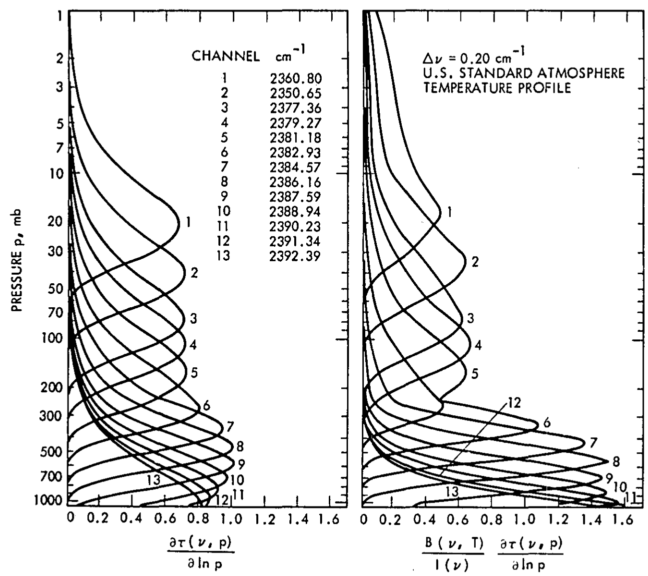


Fig. 2. 4.2-$\mu$m weighting functions and normalized integrand for $\Delta\nu$ = 0.2 cm$^{-1}$.

The soundings obtainable from such an array of narrowband channels would be ideal for use in numerical weather prediction, especially if simultaneous measurements of the same field of view could be obtained in water vapor channels and 15-$\mu$m channels. The complementary long-wave channels are necessary to obtain soundings under the normal conditions of partial cloudiness.[12,13] Practical implementation of this very narrow band-path approach, however, is difficult at present because of the high measurement accuracies required for an unambiguous interpretation of the radiance data.

The tropospheric weighting functions shown in Fig. 2 are in fact narrower than the theoretical weighting functions predicted for soundings in the wings of the lines. This is a result of strong positive temperature dependence of the coefficients $b_j$ in the troposphere. As we shall see, advantage can also be taken of the temperature dependence of the transmission to obtain narrow weighting functions even with considerably wider channel widths.

## Temperature Dependence of the Weighting Functions

The typical ratio of the half-peak pressures for the tropospheric weighting functions in Fig. 2 is about 2.6, rather than 3.4 as predicted by Eq. (9). This is a result of the aforementioned temperature dependence of the high-$J$ lines, which acts to enhance the pressure effect in the troposphere, where the temperature decreases with height, and of the linear dependence of $b_j$ on the intensities of the lines contributing to the wing absorption coefficient. At the 700-mb level of a standard atmosphere, for example, the variation of intensity with temperature has the effect of an extra factor of $p$ in the exponent at 2383 cm$^{-1}$ and a factor of $p^2$ at 2390 cm$^{-1}$. The width of a weighting function corresponding to a transmission function of the form $\exp(-cp^n)$ has a value $\Delta lnp$ at half-height (or any given fraction of its maximum height) inversely proportional to the value of $n$. This temperature-narrowing of the tropospheric weighting function suggests that effective values of $n > 1$, or even $n > 2$, can be obtained by a careful choice of considerably broader channels than those used in Fig. 2. Again the $R$-branch of the 4.3-$\mu$m $CO_2$ band is the appropriate choice of spectral region for channel selection, since it both takes advantage of the $B(T)$ effect and avoids low-$J$ isotope and $N_2O$ lines on the low frequency side of this band. In the R-branch, however, there is considerable convergence of the positions of lines with rapidly varying intensities, and appreciably narrower channel widths than those presently used are still required in order to utilize the increased capability of vertical resolution.

Figure 3 presents curves of $d\tau/dlnp$ and $Bd\tau/dlnp$ for a set of mostly contiguous triangular channels of width at half height $\Delta\nu = 2$ cm$^{-1}$. Since each of these channels contains from two to four lines, the weighting functions are of course not as narrow as those in Fig. 2. They are not broader by a factor of 2, however, despite the $p$ vs $p^2$ dependence in the transmission function exponent and the fact that $a_j$ in Eq. (7) is a linear function of the square root of the line intensities. The principal factors

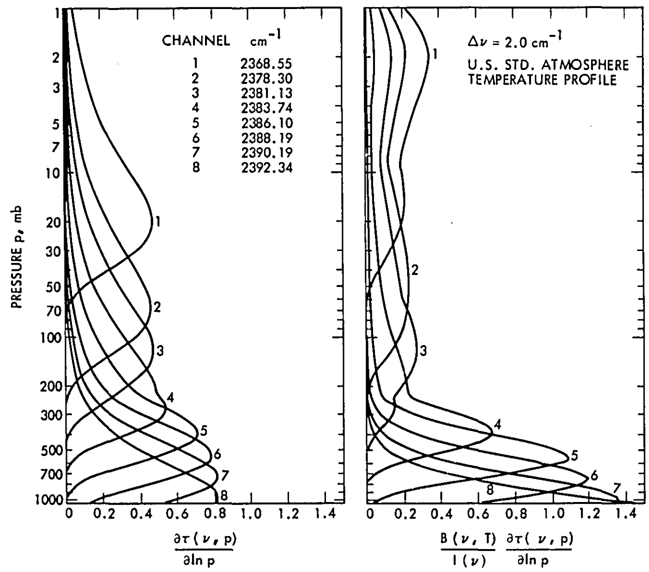


Fig. 3. 4.2-$\mu$m weighting functions and normalized integrand for $\Delta\nu$ = 2 cm$^{-1}$.

inhibiting degradation are the commonality of the $p^2$ effect of the $N_2$ continuum absorption (which, in addition has weak temperature dependence[10]) and the shifting to higher $J$ numbers, and therefore greater temperature dependence, of the spectral position corresponding to the weighting function peak pressures.

The ratio of the half-peak pressures for the tropospheric weighting functions in Fig. 3 is about 3.0, still considerably smaller than for any present temperature sounding experiment.

The upper parts of the stratospheric weighting functions, on the other hand, are appreciably broader than those in Fig. 2, since the contribution of the $N_2$ continuum is minimal and the temperature dependence acts counter to the pressure effect when the temperature increases with height.

This broadening of the stratospheric weighting functions seriously degrades the retrievability of the stratospheric thermal structure, as can be appreciated by comparison of the $Bd\tau/dlnp$ curves with those in Fig. 2. In these curves, which better indicate the resolvability, further degradation occurs in both extended wings because of the $B$ factor.

Part of this loss in retrievability can be regained by the use of channels in the 15-$\mu$m $CO_2$ band, where the temperature dependence of the stratospheric weighting functions and that of the blackbody functions are considerably reduced relative to those of the 4.3-$\mu$m channels.

## Concept of a Complete Sounding

An even more important reason for including a complementary set of 15-$\mu$m channels in a high resolution sounder is to be able to retrieve the thermal structure down to the surface in the presence of broken clouds. Chahine[12,13] has shown that such a retrieval can be obtained from simultaneous measurements in the 4.3-$\mu$m and 15-$\mu$m bands. The channel selection can be accomplished by detector arrays in the plane of a grating used in first order in the 15-$\mu$ band and in fourth order in the 4.3-$\mu$m band. If the 15-$\mu$m detectors have the same width as the 4.2-$\mu$m detectors giving the 2-$cm^{-1}$ resolution assumed for the weighting functions in Fig. 3, the spectral resolution in the 15-$\mu$m band would be 0.5 $cm^{-1}$, allowing placement in the wings of the most prominent lines and avoidance of $H_2O$ lines.

Curves of $d\tau/dlnp$ for a set of 0.5-$cm^{-1}$ channels from 15 $\mu$m to 17 $\mu$m are presented in Fig. 4. The lowest tropospheric channels are suitable for use in eliminating the effect of clouds from the 4.2-$\mu$m radiances, and the stratospheric channels are clearly better for sounding the stratosphere than the 2-$cm^{-1}$ channels at 4.2 $\mu$m; in fact, they are comparable to the 0.2-$cm^{-1}$ channels. The degradation by the $B$ factor is much less, and the high resolution is effective in utilizing the $Q$ branch to sound to considerably greater heights.

The spectral range from 15 $\mu$m to 17 $\mu$m in first order corresponds to 3.7–4.3 $\mu$m in fourth order. This would provide window channels to correct for reflected sunlight in the 4.2-$\mu$m region.[14] The influence of departure from Boltzmann equilibrium on the 4.2-$\mu$m radiances can be determined with the aid of the 15-$\mu$m stratospheric retrievals. The 1-$cm^{-1}$ resolution in second order provides several excellent windows for determining the surface and cloud top temperatures[12] and interpreting the measurements near 3.7 $\mu$m. The 1.5-$cm^{-1}$ resolution in third order will make it possible to obtain the water vapor distribution even into the stratosphere. A determination of the water vapor distribution is not only necessary in itself but also necessary to correct the 15-$\mu$m band transmittances, since the wings of water lines contribute to the opacity even if the line cores are avoided. The weighting functions in Fig. 4 were determined for a dry atmosphere; for a moist atmosphere they would sound at relatively higher levels.

The decrease of specific humidity with height in the troposphere has the effect of considerably sharpening $d\tau/dlnp$ in the $H_2O$ channels, so that moisture profiles can be obtained with vertical resolution comparable to that of the improved temperature profiles. Thus the measurements in the 4 orders would permit a detailed sounding of the troposphere and stratosphere.

A preliminary study has shown that the development of such a sounder is within the present state-of-the-art, and a design study is underway. Further details and the results of retrieval simulations will appear in a forthcoming paper.

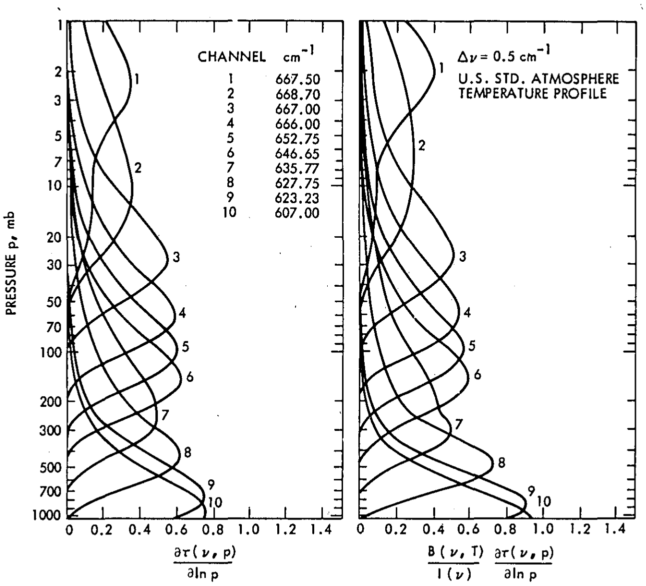


Fig. 4. 15-$\mu$m weighting functions and normalized integrand for $\Delta\nu = 0.5$ $cm^{-1}$.

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